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$$\therefore \frac{r+(z-r)\sec\theta}{u}=\frac{yz}{m}. \qquad \therefore y=\frac{m}{uz}(r-r\sec\theta+z\sec\theta)=y'.$$

The limits of θ are 0 and 2π; of z, 0 and $2r$ for total, and $r(1+\cos\theta)=z'$ to $r(1-\cos\theta)=z''$ for favorable cases; of y, 0 and $2\sqrt{(2rz-z^2)}=y''$ for total, and 0 and y' for favorable cases.

$$\therefore p=\frac{\int_0^{2\pi}\int_{z''}^{z'}\int_0^{y'}d\theta\, dz\, dy}{\int_0^{2\pi}\int_0^{2r}\int_0^{y''}d\theta\, dz\, dy}=\frac{1}{2\pi^2r^2}\int_0^{2\pi}\int_{z''}^{z'}\int_0^{y'}d\theta\, dz\, dy$$

$$=\frac{m}{2\pi^2r^2u}\int_0^{2\pi}\int_{z''}^{z'}\frac{(r-r\sec\theta+z\sec\theta)d\theta\, dz}{z}$$

$$=\frac{m}{\pi^2ru}\int_0^{2\pi}(1+\sec\theta\log\tan\tfrac{1}{2}\theta-\log\tan\tfrac{1}{2}\theta)d\theta=\frac{2m}{\pi ru}.$$

Therefore the chance of meeting one derelict out of the n is $\Sigma p=2nm/\pi ru$, and the probability that e will be encountered is

$$\Pi\left(\frac{2nm}{\pi ru}\right)=\left(\frac{2nm}{\pi ru}\right)^e.$$

CALCULUS.

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203. Proposed by S. A. COREY, Hiteman, Iowa.

Evaluate* $\int_0^\pi \frac{\sin mx\, dx}{x}$.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

$$\int_0^\pi \frac{\sin mx}{x}dx=\int_0^\pi\left(m-\frac{m^3x^2}{3!}+\frac{m^5x^4}{5!}-\frac{m^7x^6}{7!}+\text{.........}\right)dx$$

$$=m\pi\left(1-\frac{m^2\pi^2}{3.3!}+\frac{m^4\pi^4}{5.5!}-\frac{m^6\pi^6}{7.7!}+\text{.........}\right).$$

Also solved by **L. E. Newcomb.**

204. Proposed by M. E. GRABER. A. M., Heidelberg University. Tiffin, Ohio.

Required the variation of $\int Vdx$ where V is a function of x, y, $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$..... and v where $v=\int V'dx$ and V' is also a function of x, y, $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$.....

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*See Byerly's *Integral Calculus* (p. 23, Table of Integrals) Formulae 211 and 241. A solution not in the form of an infinite series would also be desirable.

*Solution by the PROPOSER.

The variation of $\int V dx$ may be divided into $\triangle u$ and $\triangle u_1$, the former arising supposing v constant, the latter from the variation of v. Thus $\triangle u = H + \int K \triangle y dx$, $\triangle u_1 = \int \frac{dV}{dv} \triangle v dx$. If the letters represent for V what their primes represent for V', $\triangle u = \int (N' \triangle y + P' \triangle p + \ldots\ldots) dx$. If $L = \frac{dV}{dv}$ and $I = \int L dx$,

$$\triangle u_1 = \int L \triangle v dx = I dv - \int I \frac{d \triangle v}{dx} dx = I(H' + \int K' \triangle y dx) - (Hi' + \int Ki' \triangle y dx)$$

(Hi, Ki denoting H' and K' when IN', IP', etc., are substituted for N', P',..... Then

$$\triangle u + \triangle u_1 = \triangle y \left(P - \frac{dQ}{dx} \ldots\right) + \triangle p \left(Q - \frac{dR}{dx} + \ldots\right) + \ldots\ldots$$

$$+ \int \left(N - \frac{dP}{dx} + \frac{d^2 Q}{dx^2} - \ldots\right) \triangle y dx + I \triangle y \left(P' - \frac{dQ'}{dx} + \frac{d^2 R'}{dx^2} \ldots\right)$$

$$+ I \triangle p \left(Q' - \frac{dR'}{dx} + \ldots\right) + \ldots + I \int \left(N' - \frac{dP'}{dx} + \frac{d^2 Q'}{dx^2} \ldots\right) dx$$

$$- \triangle y \left(IP' - \frac{dIQ'}{dx} + \ldots\right) - \triangle p \left(IP' - \frac{dIQ'}{dx} + \ldots\right) - \ldots\ldots$$

$$- \int \left(IN' - \frac{dIP'}{dx} + \frac{d^2 IQ'}{dx^2} \ldots\right) dx.$$

Also solved by G. B. M. Zerr.

205. Proposed by Z. T. JACKSQN, St. Louis, Mo.

Evaluate† $\int_0^{\frac{1}{2}\pi} \log \sin x \, dx$.

Solution by J. E. SANDERS, Reinersville, Ohio.

$$u = \int_0^{\frac{1}{2}\pi} \log \sin x \, dx = \int_0^{\frac{1}{2}\pi} \log \sin(\tfrac{1}{2}\pi - x) dx = \int_0^{\frac{1}{2}\pi} \log \cos x \, dx.$$

$$\therefore 2u = \int_0^{\frac{1}{2}\pi} (\log \sin x + \log \cos x) dx = \int_0^{\frac{1}{2}\pi} \log(\sin x \cos x) dx$$

$$= \int_0^{\frac{1}{2}\pi} \log \frac{\sin 2x}{2} dx = \int_0^{\frac{1}{2}\pi} \log \sin 2x \, dx - \frac{\pi}{2} \log 2.$$

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*See Williamson's *Integral Calculus*, Sixth Edition, p. 275.

†Byerly, *Integral Calculus*, p. 102.